Mean Temperature Difference: A Reappraisal

The derivation of the mean temperature difference in heat exchangers is based on a number of assumptions or idealizations, the most important ones being that the heat transfer coefficient is constant throughout the exchanger; that the temperature of either fluid is constant over any cross section of its nominal path, that is, complete mixing, no stratification or bypassing; and that an additional assumption for shell and tube exchangers, which has not always been fully recognized, is that within one baffle crossing the shell fluid temperature change is small with respect to its overall change, (that is, number of baffles is large). In actual exchangers, any of the above assumptions are frequently subject to various degrees of invalidation. This paper examines the effects of deviating from the first two assumptions and presents a new solution to the third.

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SCOPE

As indicated by the title, this paper examines the magnitude of potential error in evaluation of the mean temperature difference (MTD) in heat exchangers caused by departure from a number of simplifying assumptions necessary for a straightforward solution of the MTD equations. Attention in this paper is confined to the three significant ones.

1. The assumption that the overall coefficient is constant throughout the exchanger is most frequently violated, especially for fluids with steep viscosity characteristics and large temperature changes. Existing literature on this subject is scattered and confined to a few specific cases.

2. Bypassing in heat exchangers occurs if the flow splits into several branches owing to unequal flow resistances. Each stream is exposed to different conditions of heat transfer, and the mixed exit temperature can be severely distorted against idealized assumptions. While this problem has been qualitatively realized for some ideal conditions, solutions for (or even awareness of) the potentially disastrous results in industrial applications have not been properly recognized.

3. The consequences of the third assumption, that the number of baffles in a shell and tube exchanger is *large*, have never been analyzed before. The potential error is particularly apparent in a one shell pass—one tube pass exchanger, customarily considered as operating in counterflow. However, when only a few baffles are used, the flow in each baffle space is in cross flow, resulting in a possible severe MTD penalty.

CONCLUSIONS AND SIGNIFICANCE

While simplifying assumptions for solutions of the mean temperature difference (MTD) in heat exchangers are necessary for explicit solutions, most of the assumptions are violated in practical applications. This paper focuses on three assumptions considered the most frequent and most serious ones. For the first two assumptions, namely, change of the overall coefficient between hot and cold terminal temperatures and effects of fluid bypassing and unequal heat transfer, a critical review of the conditions leading to potential errors is presented. Remedial actions are recommended on a systematic basis as good as the present state of the art permits. A comprehensive review of the meager and scattered literature sources on the subject is given.

The third assumption, that the number of cross flow baffles in a shell and tube exchanger is *large*, is appraised by comparison of the conventional MTD to that of an exchanger synthesized from pure unmixed cross flow elements, all other conventional assumptions remaining valid. A lower bound to the MTD is thus established (the upper bound being the conventional MTD) over the range from no baffles at all to any arbitrary number. Interpolation formulas between the two are proposed. Although attention is confined to single-pass, TEMA type E shells with one- or two-pass tubes, the general procedure required for other configurations is apparent.

As a final comment and advice for caution, the authors would like to emphasize that use of complex, sophisticated computer programs will resolve some of the problems treated in this paper but will never become a substitute for a sound engineering judgment and individual analysis of critical cases.

Underlying all heat exchanger calculations is the basic equation

$$dQ = U(T - t)dA \qquad (1)$$

This equation is strictly valid for pointwise values of the coefficient U with respect to heat transfer surface dA and any arbitrary temperature-heat release relationship, expressed as dQ vs. temperature change. In this sense, actually all the terms of Equation (1) can be interrelated but must be then solved only on a pointwise (or stepwise) basis. Rearranging Equation (1) and integrating, we get

$$A = \int_0^Q \frac{1}{U(T-t)} dQ \tag{2}$$

If we calculate stepwise U and corresponding ΔT and plot $1/[U(T-t)]$ vs. Q, the area under such curve will represent the required surface A. A similar procedure was demonstrated in a classical paper by Colburn (1933). The pointwise calculations can get quite involved and are, for all but simple problems, practically solvable only by sophisticated computer programs, and even in such case the techniques of doing so are just emerging.

For many flow arrangements and heat transfer processes, especially in no phase change, the integration of Equation (1) can be performed under a number of restrictive assumptions. These are, in approximately decreasing order of importance:

1. The overall heat transfer coefficient U is constant throughout the exchanger.
2. For pure counterflow or cocurrent flow, the temperature of either fluid is uniform over any cross section of its path. For multishell pass and multitube pass arrangements, this applies within any shell or tube pass. In other words, the thermal history of any particle in either stream is identical to any other particle in that stream, that is, complete mixing, no stratification or bypassing. For cross flow arrangements, each fluid is considered either completely mixed or unmixed normal to the flow, and mixed or unmixed between passes.
3. For baffled shell and tube exchangers, the heat transferred in each baffle compartment is small compared to the overall; that is, the number of baffles is large.
4. The flow rate and specific heat of each fluid is constant.
5. There is no change of phase of either fluid in only a part of the exchanger. If condensation or boiling occurs, it must do so uniformly over the entire surface and in such a way that equal quantities of heat are exchanged for equal changes of the fluid temperature. This produces a linear plot of heat exchanged vs. temperature.
6. There is equal heat transfer surface in each tube or shell pass.
7. Heat losses to surroundings are negligible.

If all the above restrictive assumptions are considered valid, integration of Equation (2) for counterflow (or cocurrent flow) yields the elementary solution for the mean temperature difference Δt_m as $\Delta t_{\log}$, which becomes a function of the hot and cold terminal temperature differences D_h and D_c:

$$\Delta t_m = \Delta t_{\log} = \frac{(D_h - D_c)}{\ln(D_h/D_c)} \xrightarrow[D_h \to D_c]{} \frac{D_h + D_c}{2} \tag{3}$$

For various combinations of counterflow and cocurrent flow such as occur in multitube pass exchangers, Δt_m is obtained by means of a correction factor F:

$$\Delta t_m = F \Delta t_{\log} \tag{4}$$

The F factor formulation will be used throughout this paper as being the most illustrative one, representing directly the penalty against the best possible case of counterflow. Recalculations to the sometimes preferred presentation as number of transfer units NTU are easily possible through the following relationships:

$$F = \frac{1}{(\mathrm{NTU})\delta} \tag{5}$$

where

$$\delta = \frac{R-1}{\ln\left(\dfrac{1-P}{1-PR}\right)} \xrightarrow[R \to 1]{} \frac{1-P}{P} \tag{6}$$

However, in actual exchangers several of the above assumptions are *frequently* violated to some extent, resulting in various degrees of potential inaccuracy. The first three asumptions are considered not only the most important ones but also the most misunderstood ones, being either not at all or only fragmentarily documented. Consequently, the remaining content of this paper is devoted to review of the problem regarding deviations from assumptions 1 and 2 and presents a solution to assumption 3, which has, to the best knowledge of the authors, never been considered before.

EFFECTS OF VARIATION OF HEAT TRANSFER COEFFICIENT U

Only in rare cases will the assumption of constant U be strictly valid. A commonly used simple equation form for heat transfer coefficient suggests that for turbulent flow approximately the following relationship holds:

$$h \propto k^{0.6} c^{0.4} \mu^{-0.4} \tag{7}$$

For liquids, the k and c variations with temperature roughly offset their effects (except in extreme cases of approaching critical temperature) so that h remains mainly dependent on μ. Thus, for liquids with steep μ vs. T characteristic, h can change significantly between inlet and outlet. For gases, all properties involved generally increase with temperature, but only mildly so, and thus gas coefficients are relatively stable.

Analytical solutions to several flow arrangements exist in the literature, in all cases under assumption of linear variation of U between inlet and outlet. These are reviewed in the following.

Countercurrent Flow Exchangers

For countercurrent flow, an analytical solution was derived by Colburn (1933) as the logarithmic mean of the products $U_h D_c$ and $U_c D_h$, under the assumption that U varies linearly between the hot and cold end of the exchanger:

$$\frac{Q}{A} = \frac{(U_h D_c - U_c D_h)}{\ln(U_h D_c)/(U_c D_h)} \xrightarrow[U_h D_c \to U_c D_h]{} \frac{U_h D_c + U_c D_h}{2} \tag{8}$$

This compares to the constant U_{avg} coefficient formulation combined with $\Delta t_{\log}$ as follows:

$$\frac{Q}{A} = U_{avg} \frac{(D_h - D_c)}{\ln(D_h/D_c)} \xrightarrow[D_h \to D_c]{} U_{avg} \frac{D_h + D_c}{2} \tag{9}$$

U_{avg} can be interpreted as:

1. Arithmetic average between U_h and U_c (two calculations for U).
2. U calculated from shell stream and tube stream h values, each referred to the arithmetic average temperature of that stream (single calculation for U).
3. Colburn (1933) also derived an alternate method to Equation (8) by which a caloric temperature is defined so that if used for evaluation of U_{avg} and used in Equation (9), a correct result will be obtained.

The question obviously arises how much error is committed by using Equation (9) against Colburn's Equation (8). Expressing both equations in terms of $D_h/D_c = D' = (1-P)/(1-PR)$ and $(U_h/U_c) = U'$ and taking the ratio of Equation (8) to Equation (9), we get an expression for the error ratio or correction factor F:

$$F = \frac{2D'U'(1/D' - 1/U') \ln D'}{(1+U')(D'-1) \ln (U'/D')} \tag{10}$$

This is shown graphically in Figure 1. Values of $F < 1.0$ indicate that if we use Equation (9) under conditions of 1 or 2, the exchanger will be underdesigned, compared to the Colburn solution, Equation (8). In addition, it is of interest to make a comparison between the correct solution derived from stepwise evaluation of Equation (2) and results obtained from Equation (8) and (9) under the conditions 1, 2, or 3. For nearly linear behavior of U, Equations (8) and (9), condition 3, will be practically identical and equal to the correct solution. Also, Equation (9), conditions 1 and 2, will produce the same result and therefore the same error, compared to Equation (8). For moderately nonlinear behavior of U, the correct solution will be between that of Equation (8) and Equation (9), condition 3, the latter being frequently more accurate. There is no clear-cut preference for the simplified assumptions 1 and 2 to Equation (9), depending on which average will more closely approximate the true average of the nonlinear profile, but it appears that the condition 2 will result in lesser error for most cases, even though requiring fewer calculations. For highly nonlinear behavior of U, such as transition between laminar and turbulent flow, resort must be taken to stepwise calculation, Equation (2).

Multitube Pass Exchanger

For other than countercurrent flow, the analytical solution of the variable U problem becomes much more complicated. A rather simple solution was suggested by Sieder and Tate (1936) recommending to combine the Colburn's counterflow solution, Equation (8), with the F correction factor, based on constant U, appropriate to any particular flow configuration

$$\frac{Q}{A} = (F) f(U_h D_c, U_c D_h)_{\text{Equation (8)}} \qquad (11)$$

This general and simple procedure had the qualified approval of Bowman et al. (1940) in lieu of any better method available at that time. Indeed, this method was checked against examples presented by Gardner (1945) and Ramalho and Tiller (1965) and found to produce results within $\pm 10\%$, except in extreme cases of severe temperature approach, which one would avoid anyway owing to very low values of F. This was originally observed by Gardner (1945) and recently confirmed by Kao (1975) from a numerical analysis method for TEMA E and J types of shells.

Several specific attempts were made on the analytical solution of 1-2 exchangers with variable U. In historical sequence, Gardner (1941) presented a paper aimed primarily for exchangers with unequal surface in two tube passes. The F factor is presented as function of the ratio of $(UA)_{cf}/(UA)_{cc}$. It is suggested in the conclusion of the paper that for cases of equal tube pass surfaces, the method can also be interpreted as being applicable for linear variation of U with tube side fluid controlling. A set of F vs. P charts is presented for selected values of a parameter which can be interpreted as representing the ratio of U_h/U_c. Unfortunately, no numerical example is given. A later paper by Gardner (1945) investigated specifically the effects of variable U as a linear function of the shell stream temperature in 1-2 exchangers under the assumption of shell side film controlling. Equations and graphs are given for calculating F correction factor as functions of P, R, and U_h/U_c. A numerical example is presented for rather severe change of coefficients ($U_h/U_c = 3$) with shell side film controlling, showing that application of Equation (11) would have resulted in 10% underdesign, but neglect of the effect of variable coefficient U altogether produces a severe underdesign of 29%.

An apparently similar analysis to that by Gardner (1941) for 1-2 exchangers, assuming linear variation of U as function of the tube side temperature, was developed by Ramalho and Tiller (1965). The results are presented in a set of P vs. NTU graphs, each for a discrete value of U_h/U_c, but the mathematical formulations are left in the form of differential equations and are rather difficult to compare with other specific presentations. The authors were apparently unaware of previous work on the same subject by Gardner (1941, 1945). A numerical example is shown with $U_h/U_c = 2$. It is of interest to note that the method by Gardner (1941) produced the same result (within reading accuracy of the graphs). Using the simplified method [Equation (11)] produces for this example an overdesign of about 10%. It is also of interest to notice that the example of shell side fluid controlling resulted in underdesign when the U variation was neglected, while for tube side fluid controlling the example showed an overdesign. This should not be taken as a rule, as the effects will depend, among other things, on whether the controlling fluid is being heated or cooled.

All the above methods are based on the assumption of linear variation of U. For nonlinear variation of U, Gardner (1945) suggests an analytical expression, but the method is not developed to a practical working level. A recent and previously mentioned comprehensive investigation by Kao (1975) uses numerical solution of simultaneous nonlinear differential equations for analysis of several combinations of cases with shell and tube side fluid controlling in a 1-2N exchanger with inlet tube pass in counterflow and cocurrent flow with respect to shell side flow as well as for cases where tube side fluid is being cooled and heated. Kao's conclusions essentially support previous observations by Gardner (1941, 1945). The numerical analysis involved permits any arbitrary variation of U as well as the heat capacity c but, while useful for detailed investigation of specific cases, would probably be too cumbersome as a generalized design procedure.

A variable coefficient in various cases of cross flow arrangement was investigated by Roetzel (1974), but again the method does not appear to be sufficiently general to be considered an industrial user oriented method.

The following comments will summarize the problems connected with Δt_m determination for cases of variable U.

1. Considerable error can be committed by using $\Delta t_{\log}$ without any correction. Figure 1 can be used as an approximate measure of the potential error.

2. For counterflow exchangers, use the Colburn's logarithmic mean method [Equation (8) or Figure 1].

3. As a general method, including any flow arrangement for which a specific method does not exist, the approximation method, Equation (11), appears to be a reasonable compromise, producing results within about 10% accuracy for all but extreme temperature approach cases.

4. If greater accuracy is desired and TEMA E shell type of exchanger is used, apply the method by Gardner (1945) for shell side film controlling and the method by Gardner (1941) for tube side film controlling.

5. Above cases 2, 3, and 4 assume an essentially linear U variation. For nonlinear U variation, such as laminar-turbulent transition, stepwise procedure [Equation (2)] or numerical methods as shown by Kao (1975) must be used.

As a final comment, let us mention that the development of analytical, generalized methods for Δt_m is es-

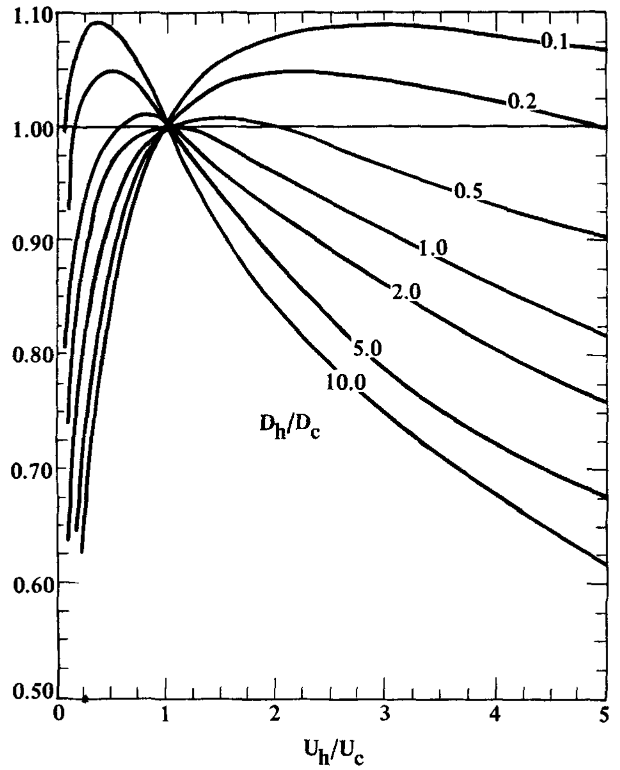


Fig. 1. Error ratio or correction factor *F* evaluated from Equation (10).

sential for manual calculations only. The almost universal use of computer programs for design of all standard types of exchangers on high-speed computers makes it now possible to employ stepwise calculations or numerical analysis methods, even though the computational complexity is considerable.

EFFECTS OF BYPASSING

Another important but frequently violated assumption for Δt_m validity is the condition that the thermal history of all fluid particles is identical (assumption 2). In principle, we are concerned here with a situation where, for reasons of unequal flow resistances, two or more streams develop within a heat exchanger and because of their flow path are not heated (or cooled) to the same degree and, in the extreme case, exchange heat *only through mixing*, which can be continuous or sudden.

For baffled shell and tube exchangers, it was shown by Tinker (1958) and later by Palen and Taborek (1969) that the shell side flow is not uniform but rather composed of the main cross flow stream and usually three identifiable bypass streams: between baffle and shell (E stream), between baffle holes and tubes (A stream), and between tube bundle and shell wall (C stream). The magnitude of these streams can vary widely depending on the geometries involved, the driving pressure differential between baffles, and, as the resistances of the individual streams are function of *Re*, the flow regime.

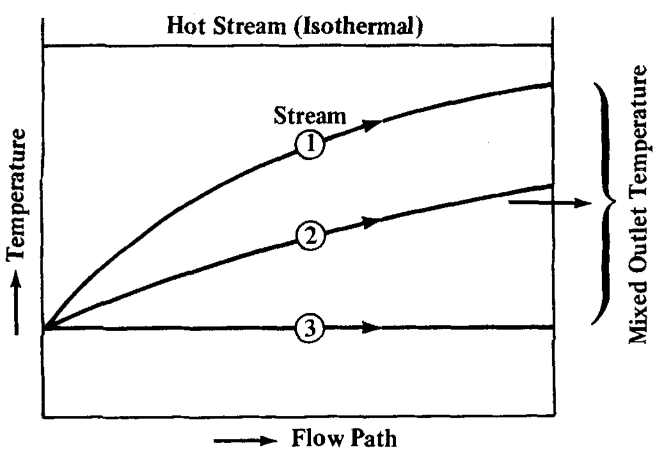


Fig. 2. Temperature change within a flow element.

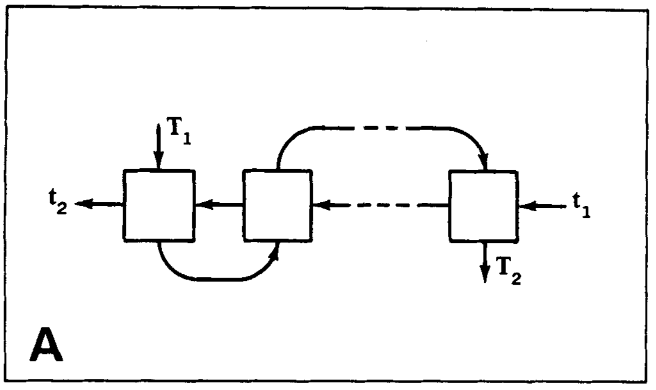
In other exchanger types, a somewhat different case of bypassing was experimentally demonstrated by Weierman et al. (1975) in flow across in-line finned tube banks, producing a dramatic error in Δt_m. Still another type of bypassing or maldistribution can occur in tube side flow for laminar cooling where part of the tubes can freeze up, as shown by Mueller (1974) and further confirmed by Kao (1975).

The bypassing problem is schematically illustrated in Figure 2. Stream 1 is fully heat transfer effective, such as the cross flow stream in shell and tube exchangers. Stream 2 is partially effective, such as the C stream, while stream 3 is completely ineffective. A mixed outlet temperature is then obtained, which shows an apparent temperature difference driving force which is much larger than the one effectively present. Thus, uncorrected bypassing will *always* result in underdesign of exchangers or, if applied for interpretation of experimental data, will result in underestimation of the true film coefficient for the fully effective stream. A secondary, but not less important, effect is, of course, the *decrease of velocity* of the effective stream, resulting in a reduced film coefficient.

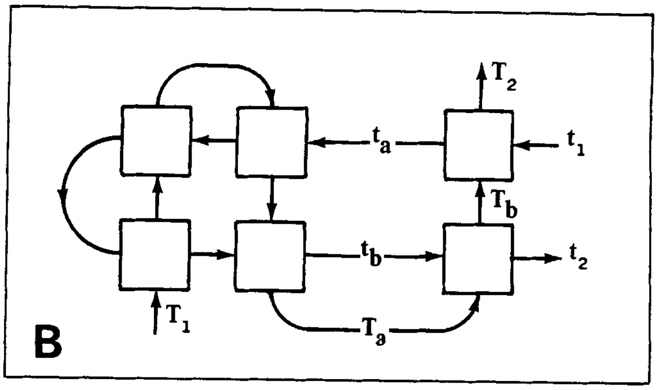
For countercurrent shell and tube exchangers, the effects of bypassing on Δt_m were first recognized by Whistler (1947) and a simplified analysis presented on a baffle-by-baffle basis. The difficulty at that time was the estimation of the bypass streams magnitude, which became more realistically possible through the work of Tinker (1958), later refined by Palen and Taborek (1969). Short (1960) also recognized the effects of bypassing Δt_m but did not develop a practically applicable method. Palen and Taborek (1969) developed a qualitative empirical method for shell and tube exchangers in the form of Δt_m penalty factor which is function of the E stream relative magnitude and the ratio of the terminal temperature approach *to the shell side temperature rise*. The effect of Reynolds number is also observed to be of great importance, as it will affect the degree of mixing.

Fischer and Parker (1969) developed an analytical method for the effect of bypassing on Δt_m in a 1-2 exchanger. Here, the bypass stream is considered as the sum of the A, C, and E streams and as being thermally fully ineffective. Complete mixing is assumed to occur at each baffle crossing. The analysis is not presented in the usual dimensionless terms, but a number of graphs for the correction factor *F* as function of *P* for discrete combinations of bypass fractions and number of baffles crossed is shown. While the simplified assumptions were probably necessary in order to permit a manageable solution, their validity can be somewhat questioned. For example, the A stream (tube-to-baffle hole) is relatively *effective and mixes immediately* with the cross flow stream; the C stream is at least partially effective, or can be made so by use of sealing strips, and also mixes continuously with the cross flow stream. The most ineffective E stream (baffle to shell) is considered independent of *Re*, which is contrary to what was observed from experimental data, such as shown by Bell (1963) and Palen and Taborek (1969).

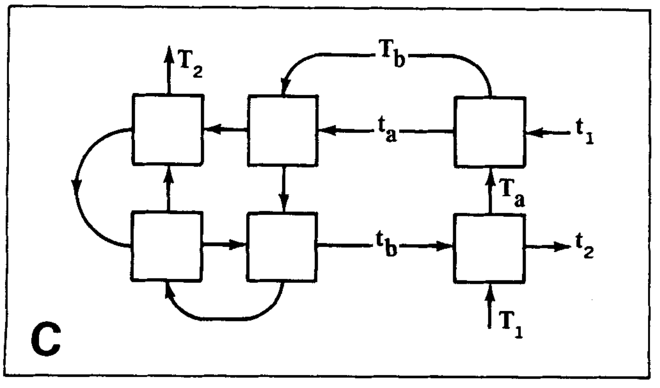
A generalized analytical solution for an elementary



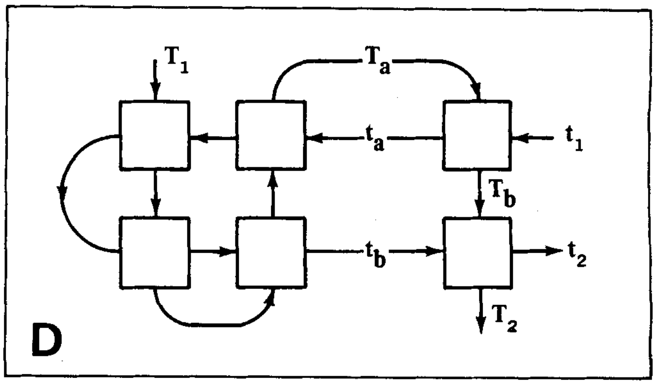
A 1-1 counterflow



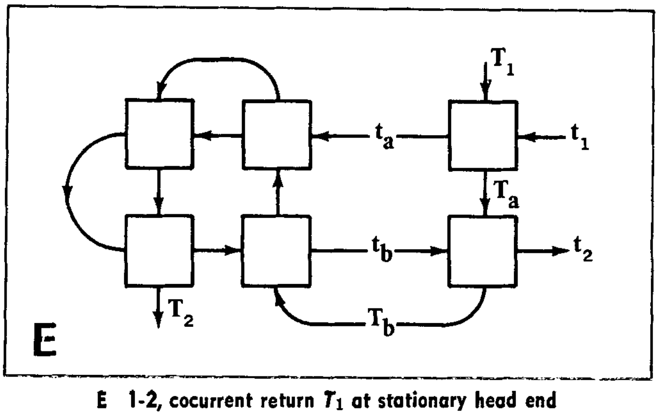
B 1-2, counterflow return T_1 at return end



C 1-2, countercurrent return T_1 at stationary head end



D 1-2, cocurrent return T_1 at return end



E 1-2, cocurrent return T_1 at stationary head end

Fig. 3. Synthesis of simulated 1-1 and 1-2 shell and tube heat exchangers from identical basic cross flow elements.

system of two streams with arbitrary mixing was presented by Bell and Kegler (1975) and compared to data by Weierman et al. (1975). It is suggested that this type of analysis will be necessary for realistic solution of the bypass problem in tube banks and shell and tube exchangers as well. However, a much more systematic set of data than presently available would have to be generated, including variation of all the basic parameters.

In summary, there is no generalized method available in published literature which would dependably account for the effects of bypassing in tube banks or shell and tube exchangers. In the latter, the effective cross flow stream for well-designed exchangers varies from about 70% in turbulent flow to as low as 20% in deep laminar flow. Thus, the real problem is the combination of effects on both temperature difference as well as the true effective film coefficient. Similar to the conclusion of the previous chapter, it appears that a stepwise calculation procedure on fast digital computers may be the only right answer, assuming the availability of methods, such as shown by Palen and Taborek (1969), to predict the magnitude of the bypass streams and their respective heat transfer effectiveness.

EFFECTS OF SMALL NUMBERS OF BAFFLES

The following discussion and derivation is basically a parametric study in the sense that all other conventional assumptions are considered valid while only one, in this case assumption 3, is changed and its effects observed by comparison to those of the original. An assumption customarily made in deriving a mean temperature difference (MTD) correction factor F for shell and tube heat exchangers is that the temperature of the shell fluid may be considered uniform over any cross section normal to the shell axis. This assumption is a good approximation to the physical situation only to the extent that the temperature change in a single compartment between adjacent baffles is a small fraction of the total temperature change of the shell fluid through the exchanger. With segment-cut baffles so disposed as to encourage cross flow of the shell stream over all tube passes in alternating directions in successive baffle compartments, validation of the assumption requires a large number of baffles per shell in either single shell pass, single tube pass (1-1), or single shell pass, two tube pass (1-2) exchangers. Limitations of time and space confine attention here to exchangers of the types described above.

At least four questions are raised by the preceding paragraphs:

1. How large is a *large number* of baffles in that context?
2. What MTD correction is required for a lesser number?
3. Does such correction remain independent of shell fluid flow direction in 1-2 exchangers as it does when the number of baffles is large?
4. On what basis can the foregoing questions be answered quantitatively and conservatively?

Basis for Analysis

Taking the last question first, we propose to consider a 1-1 exchanger with (X-1) cross flow baffles as an array of X geometrically identical pure crossflow exchangers arranged in series counterflow, as shown in Figure 3*a*. (The cocurrent series array is of little interest in practice.) A 1-2 exchanger, by similar reasoning, is considered as a much more complex array of $2X$ identical pure cross flow elements (for X-1 baffles), as shown in Figures 3*b*, *c*, *d*, and *e*. The four alternative arrays cover the cases where the shell stream at the tube return end of the shell crosses the tube bundle counter to the tube flow direction in the return head (or *U* tubes), Figures 3*b* and *c*, or in the same direction, Figures 3*d* and *e*. In either situation, a distinction must be maintained between even and odd numbers of shell stream cross flow passes, at least until X increases to such a value that the need for such distinction clearly no longer exists. At such a point (or number of cross flow passes), *large number* will have been defined in response to Question 1 above.

In all cases, the entire shell stream is first assumed to pass in uniformly distributed cross flow through each component element of the array; that is, there are no axial components of the shell stream past effective surface and no bypassing to complicate what is intended to be a *lower bound* solution for the MTD correction factor.

It is necessary also to specify the type of cross flow involved within each element and the mixing effects of passage from one element to the next. It is assumed that both streams are unmixed *within* an element (as appropriate to widely spaced baffles) and that the number of tube layers crossed per element is sufficiently high (say > 6) to justify use of pure unmixed cross flow thermal relationships for each element. It is further assumed that both streams are completely mixed *between* elements. (Actually, the tube stream is unmixed between elements except in the return head of 1-2 exchangers, and the shell stream between elements alternates from mixed, as it turns and passes through a baffle opening, to unmixed, as it crosses from one tube pass element to the other in 1-2 exchangers).

Stevens et al. (1957) obtained results for two and three cross flow exchangers in series-counterflow, unmixed between elements on one stream and mixed on the other. Comparison of these results with those for *both* streams mixed between elements and consideration of a discussion by Landis support the authors' conclusion that although some slight sacrifice of conservatism is involved, the foregoing paragraphs constitute a suitable quantitative and conservative basis for analysis in response to Question 4 above.

1-1 Exchangers with Cross Flow Baffles

For an array of X identical heat exchangers connected in series-counterflow on both streams, Figure 3*a*, Bowman (1936) has shown that the MTD correction factor F for the array as a whole is identical to that F' of any single element of the array and that

$$P' = \frac{1 - \left(\dfrac{1-P}{1-PR}\right)^{1/X}}{1 - R\left(\dfrac{1-P}{1-PR}\right)^{1/X}} \xrightarrow[R\to 1]{} \frac{P}{[X - P(X-1)]} \tag{12}$$

where P and P' apply, respectively, to the array as a whole and to any single element.

An explicit form F' equation for unmixed-unmixed cross flow of considerable accuracy is available in an ingenious general approximation by Roetzel and Nicole (1975). Figure 4, which has a much denser population of curves of constant R than any previously published for unmixed cross flow, is based on computer solution of their general equation for MTD correction factors, using the appropriate constants provided in their paper. The accuracy is very good for $F > 0.7$ and $R > 0.4$. Figure 5 shows, for $R = 1$, the variation of F_x with P and X as calculated from Equation (12) and Figure 4.

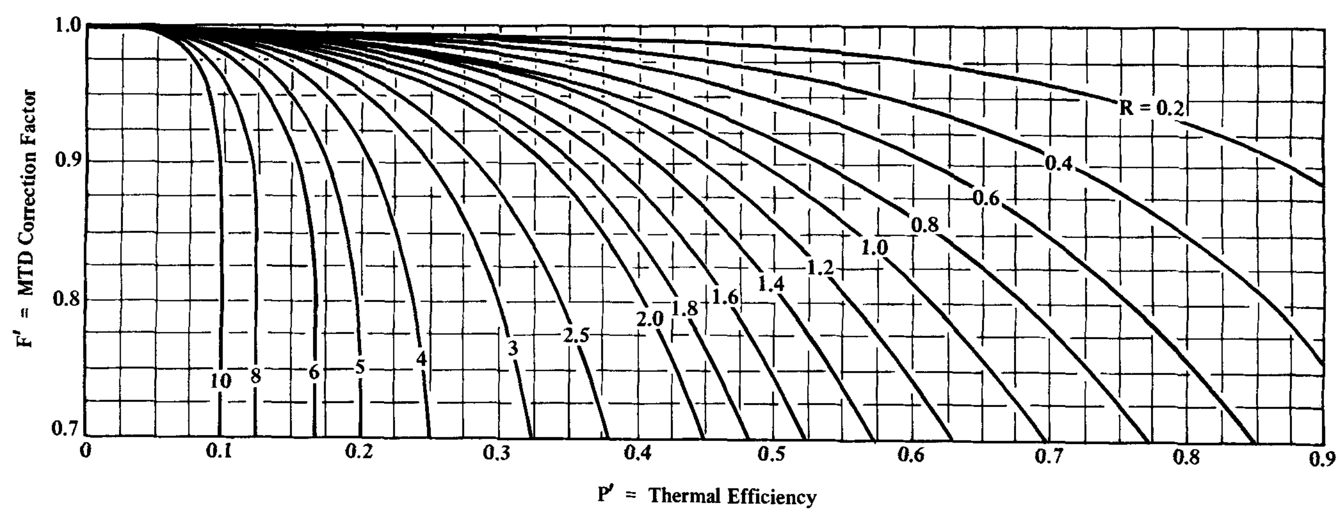


Fig. 4. MTD correction factors for cross flow heat exchangers with both streams unmixed; calculations.

Equation (12) could, of course, be solved for P instead of P'. If this is done, it is then possible, with Figure 4 as a basis, to construct separate charts of F_x vs. P and R for any desired values of X. This is not an attractive alternative to the use of one chart and one simple equation.

Conclusions on 1-1 Cross Flow Baffled Counterflow Exchangers

The unattractive alternative referred to above was pursued to a point where it became obvious that *there is no cutoff point defined solely in terms of* X beyond which the customary value, $F = 1$, for counterflow may be used with impunity. As in all MTD correction factor charts, there are areas where F_x is satisfactorily close to unity, but they remain strong functions of P and R, as well as X. The following very simple procedure, however, defines the minimum number of bundle crossings $X_{\min}$ required to provide an MTD correction factor F_x greater than 0.98. If Figure 4, or an equivalent table or equation, is entered with P instead of P' and the resulting F' designated as $F'(P, R)$, then

$$X_{\min} = 1 + 20[1 - F'(P, R)] \qquad (13)$$

and F_x may be taken as 1.0 without further ado if the actual X equals or exceeds this number. This relation is empirical rather than derived and, for the moment at least, is not thoroughly understood. It does, nevertheless, provide the answer to Question 1 for 1-1 counterflow exchangers.

If X is less than the value obtained from Equation (13), the appropriate procedure to answer Question 2 has already been furnished in the paragraphs immediately preceding these conclusions. With the use of Equation (12) and Figure 4, the procedure is not onerous for hand calculations and not at all so for computerized solutions.

Some thought must also be given to the fact that axial flow of the shell stream has been assumed negligible in the derivation for F_x. Without going as far as to recommend it, the authors suggest that the following equation provides a reasonable approximation to the true situation:

$$F = F_x + \frac{N_o}{N}(1 - F_x) \qquad (14)$$

where N is the total number of tubes in the tube bundle, N_o is the number of tubes in one baffle opening and F is the MTD correction factor appropriate to these quantities in combination with P, R, and X. This amounts to a primitive interpolation between the upper bound solution ($F = 1$) and the lower bound solution ($F = F_x$).

1-2 Exchangers with Cross Flow Baffles

No such simple relation as Equation (12) is to be found for 1-2 exchangers synthesized from cross flow elements as shown in Figures 3*b*, *c*, *d*, and *e*, even when both streams are considered mixed between adjacent elements. Nevertheless, MTD correction factors for the exchanger as a whole can be determined by use of the relations utilized previously by Gardner (1941, 1942, 1945):

$$F_x = \frac{\ln\left(\dfrac{1-P}{1-PR}\right)}{(R-1)B} \xrightarrow[R\to 1]{} \frac{P}{(1-P)B} \qquad (15)$$

$$B = \frac{UA}{wc} = 2B'X \qquad (16)$$

$$B' = \frac{UA'}{wc} = \frac{B}{2X} \qquad (17)$$

$$P = f(P', R, X) \qquad (18)$$

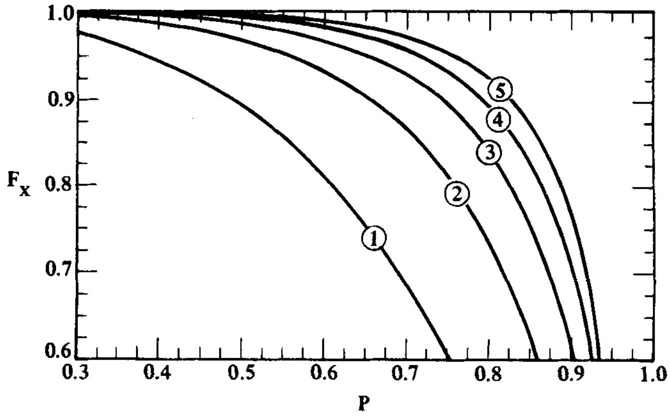


Fig. 5. MTD correction factors for simulated 1-1 exchangers with R = 1, showing effects of number of shell crossflow passes (circled numbers).

$$P' = f(B', R) \qquad (19)$$

The last of these is simply a graph, table, or equation providing values of P' for an unmixed cross flow exchanger when B' and R are given. Table 1 shown in the paper by Stevens et al. (1957) is used for this purpose here, supplemented on occasion by the Roetzel and Nicole (1975) reference.

Once an explicit equation for the relationship indicated as Equation (18) is provided, the procedure is simple, working the equations in reverse order. For numerical evaluation of F_x, the details of the procedure are given by Gardner (1942).

The development of a suitable form of the relationship indicated by Equation (18), while straightforward, is not at all simple. With reference to any one of Figures 3*b*, *c*, *d*, or *e*, it involves building up the exchanger from left to right block by block, each block consisting of two vertically stacked elements constituting one cross flow pass of the shell stream. Regardless of the stage of completion of the built-up exchanger under consideration, it may be viewed at that stage as the combination of one accumulation of blocks to the left characterized by P_{x-1} and a single block to the right consisting of two elements, each characterized by P'. The eight temperatures, T_1, T_a, T_b, T_2, and t_1, t_a, t_b, and t_2, when combined in appropriate temperature difference ratios to define P', P_{x-1}, P, and R, are sufficient to define P also in terms of R and X, but only by iteration from $x = 2$ to $x = X$.

The net results of the foregoing procedures, which are too tedious and space consuming to detail here, are two sets of recurrence formulas, one for counterflow in the left-hand (return) block, the other for concurrent flow. Each set requires the alternate use of equations for P_x for even and odd x until the desired value X is reached. Fortunately, these equations are the same whether the shell fluid enters at the stationary head end or the return end of the shell. Thus, no distinction need be made between the MTD correction factors for Figures 3*b* and *c*, nor between those of Figures 3*d* and *e*. The answer to Question 3, posed earlier, is that other things (including return flow orientation and X) being equal, the correction factor is independent of shell flow direction in 1-2 exchangers.

Counterflow Return

$$P_1 = \left[\frac{1 - \left(\dfrac{1-P'}{1-P'R}\right)^2}{1 - R\left(\dfrac{1-P'}{1-P'R}\right)^2}\right] \xrightarrow[R\to 1]{} \frac{2P'}{(1+P')} \qquad (20)$$

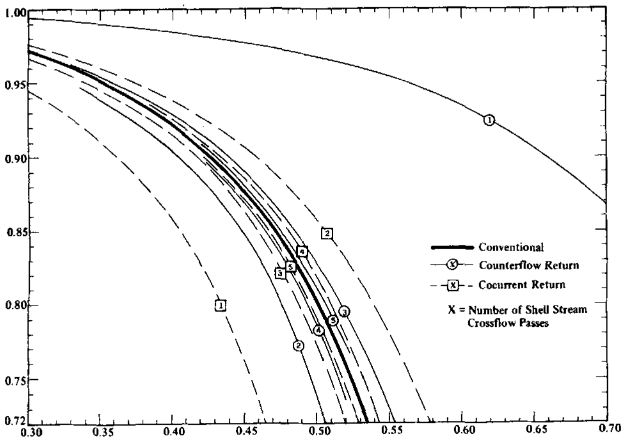


Fig. 6. MTD correction factors for simulated 1-2 exchangers with $R = 1$, showing effects of number of shell cross flow passes and orientation of tube return shell flow.

$$P_x = P' + [1 - P'(R+1)][P_{x-1} + Y_x(1 - P_{x-1})] \quad (21)$$

where

$$Y_x = P'\left(\frac{1 - P_{x-1}}{1 - P'P_{x-1}R}\right) \quad (22)$$

for even x

or

$$P_x = Y_x + (1 - Y_x)[P' + P_{x-1}[1 - P'(R+1)]\,] \quad (23)$$

where

$$Y_x = P'\left\{\frac{(1 - P'R)(1 - P_{x-1}) + P'P_{x-1}R}{1 - P'R[P' + P_{x-1}[1 - P'(R+1)]\,]}\right\} \quad (24)$$

for odd $x > 1$

Cocurrent Return

$$P_1 = P'[2 - P'(R+1)] \quad (25)$$

$$P_x = \frac{1}{R} - \frac{Y_x}{P'R}\{1 - R[P' + P_{x-1}[1 - P'(R+1)]\,]\} \quad (26)$$

where

$$Y_x = \frac{P'(1 - P'R)}{\{1 - P'R[P' + P_{x-1}[1 - P'(R+1)]\,]\}} \quad (27)$$

for even x

or

$$P_x = P' + \frac{[1 - P'(R+1)]\{P' + P_{x-1}[1 - P'(R+1)]\}}{(1 - P'P_{x-1}R)}$$

for odd $x > 1$ (28)

Numerical Results for 1-2 Exchangers

Either of the foregoing two sets of equations is solved iteratively for P by starting with Equation (20), for example, to determine P_1, using this result in Equation (21) with $x = 2$ to find P_2, using this result in Equation (23) with $x = 3$ to find P_3, and so on, alternating between Equations (21) and (23) until the required value of X is reached. This, then, gives the value of P required in the procedure, that is, the numerical solution of Equation (18) for counterflow return. For cocurrent return, substitute Equations (25), (26), and (28) for (20), (21), and (23).

Values of F_x have been determined as a function of P at $R = 0.2$, 1, and 5, $X = 1$-5 inclusive and for both counterflow and cocurrent return blocks. For clarity of presentation, these are displayed on magnified scale for that region of a correction factor chart normally of interest, that is, $F_x > 0.75$, and for $R = 1$ only. This is done in Figure 6 over the range $0.7 > P > 0.3$.

The heavy central line in the family of curves shown is the customary line for 1-2 exchangers with $R = 1$ and a *large* number of baffles. The uppermost (solid) line is that for a single crossing of a two-pass bundle with counterflow return, a normal arrangement, but not often used in cylindrical shells. The lowermost (dashed) curve is also for one shell crossing but with concurrent return which, by itself, is never used. Reading inward from either of these extremes, it is seen that increasing the number of shell stream crossflow passes by unit increments successively collapses the two curves with the same values of X closer to the heavy line for large X and that, at constant P, they are very nearly equally spaced above and below it for $X > 1$. This variance, as a percentage of the conventional F_∞ for 1-2 exchangers, is summarized in Table 1 for a minimum acceptable F of 0.75 for $R = 1$ and for both 0.75 and 0.85 for $R = 0.2$ and 5. ($F_\infty = 0.75$ at $R = 1$ and $F_\infty = 0.85$ at 0.2 or 5 are compatible with an acceptability criterion of $P_{\text{opt}} = 0.9P_{\text{max}}$.)

Conclusions on Cross Flow Baffled 1-2 Exchangers

The equations given above generate both an upper bound and a lower bound solution for F_x, as shown in Figure 6 and Table 1. The higher value occurs only for *two combinations of X and return flow orientation-counterflow return and odd X or cocurrent return and even X.* The lower value is obtained with counterflow return and even X or with concurrent return and odd X. This somewhat complicates the reply to the questions initially raised.

The authors' answer to Question 1 is that the large number sought, at and above which the conventional 1-2

TABLE 1. PERCENT VARIANCE FROM CONVENTIONAL F_∞

R	1.0	0.2 and 5.0	
X \ F_∞	0.75	0.75	0.85
3	±4.9	±6.9	±2.6
4	∓2.9	∓4.5	∓1.6
5	±1.9	±3.1	±0.9

Upper sign for counterflow return.
Lower sign for cocurrent return.

correction factors may be used with impunity, is $X = 5$ (4 baffles). The basis, as before, is that a discrepancy of ~2% from the customary F_∞ is, for practical purposes, negligible.

Question 2, what to do when $X < 5$, is subject to a facile response which, most things considered, would be the authors' preference. *In general, avoid designing 1-2 exchangers with less than four cross flow baffles.* Nevertheless, if the higher value of F_x is justified by the discussion at the start of these conclusions, it is conservative to use F_∞ regardless of X. If, however, baffle spacing and piping layout requirements conspire to insist upon the lower value of F_x, that lower value should be used, as calculated from the appropriate equations or estimated from the limited data of Table 1.

Question 3 has already been answered. The shell flow direction is immaterial, other things being equal, just as it is for F_∞.

Finally, as for 1-1 counterflow exchangers, an interpolation formula is required to compensate for oversimplification in the assumption of no axial flow components of the shell stream over effective surface. The authors suggest that

$$F = F_x + \frac{N_o}{N}(F_\infty - F_x) \tag{29}$$

similar to Equation (14) for counterflow, but with F_∞ replacing 1.0 in recognition of the fact that only half the axial flow of the shell stream is counter to the tube flow direction, the other half being cocurrent.

Summary on Cross Flow Baffled 1-1 and 1-2 Exchangers

The effect of number of cross flow baffles on the MTD in 1-1 and 1-2 TEMA type E heat exchangers has been studied using all but one of the conventional assumptions listed. The exception, assumption 3, states that the number of baffles is large, which permits analysis for temperature variation of both streams as though the shell flow were purely axial in direction. The fewer the baffles per shell, the less valid such analysis must be.

Accordingly, the present analysis is based on simulating the exchangers as assemblages of pure cross flow elements. Since the true physical situation must lie between this extreme and the conventional one, simple interpolation formulas are proposed in Equations (14) and (29).

For nominally counterflow 1-1 exchangers, the results can be quite dismaying, especially to those (most of us) whose conditioned reflex to the word counterflow is a Pavlovian $F = 1$; no tendency to salivate has been observed. For example, such an exchanger without tubes in the baffle openings designed for $R = 1.0$, $P = 0.8$, and $X = 4$ has an MTD correction factor of ~0.89, Equation (13) shows that at least eleven bundle crossings would be required in order to justify the use of logarithmic MTD without correction.*

By contrast, 1-2 exchangers may be designed with the customary MTD correction provided only that $X \geqq 5$. The penalty for lesser numbers is nonexistent under some circumstances but not severe in any practical case, as previously discussed.

As a final observation, the discomforting thought occurred to the authors that at least some data used for shell side coefficient correlations in the past were obtained from units having only very few baffles in a 1-1 exchanger, in which case invariably the assumption of straight counterflow was made. The potential errors are difficult to assess and are beyond the scope of this paper, but it is hoped that at least future researchers in this area will derive a warning message from the results presented here.

* Since the presentation of this paper, Caglayan and Buthod (1976) have published MTD correction factors for 1-1 cross flow baffled exchangers obtained by computer calculations on a model identical to the authors' except that the tube stream is considered unmixed throughout the exchanger, as is usually the case. A limited spot check indicates that at points where the authors' values of $F_x = 0.8$, they are approximately 5% higher than shown by Caglayan and Buthod (1976) and, at 0.85, 3%. The curve for $R = 1$ in Figure 6 (for $X = 3$) in the same paper is puzzling in that it does not agree with that presented by Stevens et al. (1957) and that it crosses the curve for $R = 1$ in Figure 7 (for $X = 4$). The values of Stevens et al. (1957) are used for comparison in this case.

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NOTATION

- A = total surface per shell
- A' = surface per basic element = A/X for 1-1 or $A/2X$ for 1-2 exchangers
- B = (UA/wc) = number of transfer units (NTU) per shell
- B' = (UA'/wc) = NTU per basic element
- C = heat capacity of shell fluid
- c = heat capacity of tube fluid
- d = differential operator
- D = $(T - t)$ = temperature difference between shell and tube streams
- F = $(\Delta t_m/\Delta t_{\log})$ as defined by Equation (4)
- F_x = $(\Delta t_m/\Delta t_{\log})$ for an exchanger synthesized from pure cross flow elements
- F' = $(\Delta t_m/\Delta t_{\log})$ for a basic cross flow element
- $f()$ = a function of the quantities within ()
- h = fluid film heat transfer coefficient
- N = number of tubes per shell
- N_o = number of tubes in a single segment baffle window
- P = $(t_2 - t_1)/(T_1 - t_1)$ for the exchanger as a whole
- P' = $(t_{\text{out}} - t_{\text{in}})/(T_{\text{in}} - t_{\text{in}})$ for a basic element
- P_x = $(t_{\text{out}} - t_{\text{in}})/(T_{\text{in}} - t_{\text{in}})$ for a portion of the exchanger consisting of x baffle compartments adjacent the tube return head
- Q = heat exchanged per unit time
- R = $(T_1 - T_2)/(t_2 - t_1) = wc/WC$
- Re = Reynolds number
- T = temperature of shell fluid
- T' = D_h/D_c = ratio of hot/cold terminal temperature differences
- t = temperature of tube fluid
- U = overall heat transfer coefficient
- U' = (U_h/U_c)
- W = mass flow of shell stream per unit time
- w = mass flow of tube fluid per unit time
- X = number of tube bundle crossings per shell
- Y_x = $f(P', P_{x-1}, R)$ defined by Equations (22), (24), or (27)
- Δt_m = mean temperature difference (MTD) between streams
- $\Delta t_{\log}$ = counterflow logarithmic MTD
- μ = viscosity of a fluid stream

Subscripts

- avg = average value of a quantity (defined in various ways)

a, b = intermediate points along stream
c = cold end of counterflow exchanger
cc = cocurrent tube pass
cf = counterflow tube pass
h = hot end of counterflow exchanger
max = maximum obtainable
opt = economic optimum
1 = inlet of a stream
2 = outlet of a stream

LITERATURE CITED

Bell, K. J., *Bull. No. 5*, Univ. Del. Eng. Exp. Station (1963).

———, and W. H. Kegler, "Analysis of Bypass Flow Effects in Tube Banks and Heat Exchangers," AIChE No 7, AIChE-ASME 15th Natl. Heat Transfer Conf., San Francisco, Calif. (1975).

Bowman, R. A., "Mean Temperature Difference Correction in Multipass Exchangers," *Ind. Eng. Chem.*, **28**, No. 5, 541 (1936).

———, A. C. Mueller, and W. M. Nagle, "Mean Temperature Difference in Design," *Trans. ASME*, 283 (1940).

Caglayan, A. N., and P. Buthod, "Factors Correct Air-Cooler and S&T Exchanger LMTD," *The Oil and Gas Journal*, 91 (1976).

Colburn, A. P., "Mean Temperature Difference and Heat Transfer Coefficient in Liquid Heat Exchangers," *Ind. Eng. Chem.*, **25**, No. 8, 873 (1933).

Fisher, J., and R. O. Parker, "New Ideas on Heat Exchanger Design," *Hydrocarbon Processing*, 147 (1969).

Gardner, K. A., "Variable Heat Transfer Rate Correction in Multipass Exchangers, Shell-Side Film Controlling," *Trans. ASME*, 31 (1945).

———, "Mean Temperature Difference in Unbalanced-Pass Exchangers," *Ind. Eng. Chem.*, **33**, No. 10, 1215 (1941).

———, "Mean Temperature Difference in an Array of Identical Exchangers," *ibid.*, **34**, No. 9, 1083 (1942).

Kao, S., "Analysis of Multipass Heat Exchangers with Variable Properties and Transfer Rate," *J. Heat Transfer*, 509 (1975).

Mueller, A. C., "Criteria for Maldistribution in Viscous Flow Coolers," Trans. of the 5th Intl. Heat Transfer Conf., Paper HE1-4, V, 170, Tokyo, Japan (1974).

Palen, J. W. and J. Taborek, "Solution of Shell Side Flow Pressure Drop and Heat Transfer by Stream Analysis Method," *Chem. Eng. Progr. Symposium Ser., No. 29*, **63**, 53 (1969).

Ramalho, R. S., and F. M. Tiller, "Improved Design Method for Multipass Exchangers," *Chem. Eng.*, 87 (1965).

Roetzel, Wilfried, "Heat Exchanger Design with Variable Transfer Coefficients for Crossflow and Mixed Flow Arrangements," *Intern. J. Heat Mass Transfer*, **17**, 1037 (1974).

———, and F. J. L. Nicole, "Mean Temperature Difference for Heat Exchanger Design—A General Approximate Explicit Equation," *J. Heat Transfer*, 5 (1975).

Short, B. E., "Shell-and-Tube Heat Exchangers: Effect of By-Pass and Clearance Streams on the Main Stream Temperature," ASME Paper No. 60-HT-16, ASME-AIChE Heat Transfer Conf., Buffalo, N.Y. (1960).

Sieder, E. N., and G. E. Tate, "Heat Transfer and Pressure Drop of Liquids in Tubes," *Ind. Eng. Chem.*, **28**, No. 12, 1429 (1936).

Stevens, R. A., J. Fernandez and J. R. Woolf, "Mean Temperature Difference in One, Two and Three-Pass Crossflow Heat Exchangers," *Trans. ASME*, **79**, 287 (1957).

Tinker, T., "Shell-Side Characteristics of Shell-and-Tube Heat Exchangers—A Simplified Rating System for Commercial Heat Exchangers," *ibid.*, **80**, 36 (1958).

Weierman, C., J. Taborek, and W. J. Marner, "Comparison of the Performance of Inline and Staggered Banks of Tubes with Segmented Fins," paper presented at the AIChE-ASME 15th Natl. Heat Transfer Conf., San Francisco, Calif. (1975).

Whistler, A. M., "Effect of Leakage Around Cross-Baffles in a Heat Exchanger," *Petrol. Refiner*, **26**, No. 10, 114 (1947).

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Dispersion in Layered Porous Media

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Experimental data on directional dispersion coefficients in transversely isotropic media, a special case of anisotropic media, were determined in models consisting of alternating layers of sintered glass and unconsolidated glass beads. Values of appropriate parameters to relate the velocity to the directional dispersion coefficient were determined for a variety of permeabilities and angles. Sensitivity of these parameters with respect to angle was determined, and results are presented which will permit estimate of when this effect is significant.

SCOPE

Porous media occur in many areas of chemical engineering, petroleum engineering, and hydrology. The usual difficulties with characterization of porous media occur because of their nonuniformity, heterogeneity, and anisotropy. Nonuniformity and heterogeneity are concerned with spatial variations of the media; anisotropy is concerned with variation with angle. To treat a general anisotropic medium is difficult; therefore, some special case of anisotropy is usually treated.

In this paper, we treat dispersion in transversely isotropic porous media, which are a special subset of anisotropic porous media. Transversely isotropic media characterize layered systems. Most petroleum and water reservoirs are layered. Many packed beds are packed in layers. The objective was to characterize the dispersion (mixing) of a fluid as it flows through such a medium as it relates to the angle the velocity makes with axis of isotropy and the velocity of flow. To take a single example, such knowledge is extremely important in the production of petroleum from underground deposits when this production is being accomplished by miscible displacement, that is, displacement of petroleum from the rock formation using a miscible fluid. The significant theory leading up to the treatment in this article is reviewed, and here we give results which permit the directional dispersion to be related both to velocity and to angle.

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